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ON SOME JOINT PROBABILITIES USEFUL IN
MIXED ACCEPTANCE SAMPLING

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1. INTRODUCTION

1.1. Scope

Mixed acceptance sampling plans, i.e. sampling schemes combining constituent
variables and attributes results in a double sampling procedure, provide con-
siderable advantage over more conventional plans in certain applications. This
has been pointed out by Dodge [1] and Bowker and Goode [2], among others. Such
plans have not, however, found application in proportion to their merit, partly
because methods of computing the probabilities necessary for the evaluation of
the operating characteristics of such plans for all attributes acceptance numbers
have not been developed.

Of particular importance is the case involving a single specification limit,
known standard deviation, assuming an underlying normal distribution of product,
when the results of the variables and attributes constituents of the plan are
not kept independent. At present the only detailed investigation in this area
appears to be the pioneering work of Gregory and Resnikoff [3]. Savage [4] points
out that Gregory and Resnikoff were able to find feasible computing techniques for the
necessary probabilities for this case only when c = 0, a considerable restriction.
This report includes an alternative derivation of the necessary probabilities when
c = 0 and then presents a derivation of the probabilities for c > 0, together with

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[1]The material in this report is based in part on work done in preparation of
a doctoral dissertation at Rutgers - The State University.

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a scheme for readily computing these probabilities.

1.2. The Mixed Acceptance Sampling Plan

Mixed acceptance sampling plans are of two types, "independent" and "dependent Independent plans are those in which the results of the variables and attributes components are kept independent. Such plans have been discussed by Bowker and Goode [1], Gregory and Resnikoff [3], and Schilling [5]. The values of probability of acceptance, P_a, of such plans are readily determined through the use of existing tables and procedures. Probably of more importance, both theoretically and practically, are dependent plans, those in which the variables and attributes results of the procedure are combined in such a way as to make them dependent. Attention will be directed here to dependent plans involving a single specification limit, known standard deviation, and the assumption of an underlying normal distribution of product. For reasons of symmetry, only an upper specification limit will be considered.

Let:

n_1 = the first sample size.

A = variables acceptance limit such that if the sample mean, $\bar{x} \le A$ the lot is accepted[2].

n_2 = second sample size.

c = attributes acceptance number.

There are a number of procedures for carrying out a dependent mixed plan. A typical one is that originally proposed by Bowker and Goode [1] which can be summarized as follows:

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[2]Of the several methods of specifying the variables constituent of known-standard-deviation (σ) variables plans, designation by sample size (n_1) and acceptance limit on the sample average (A), simplifies the notation somewhat. Note that A = U-kσ for upper specification limit U and customary variables acceptance factor k; also that A = (K_p - k) in the notation of Gregory and Resnikoff [3].

1. Determine parameters of plan: n_1, A, n_2, c.

2. Take a random sample of size n_1 from the lot.

3. Determine the sample average ($\bar{x}$) of this first sample and,
   a. If $\bar{x} \leq A$ accept the lot and cease inspection.
   b. If $\bar{x} > A$ continue inspection as in 4.

4. Take a second sample of size n_2 from the lot and determine the number of defectives m contained in the total combined sample of $n_1 + n_2$ and,
   a. If $m \leq c$ accept the lot and cease inspection.
   b. If $m > c$ reject the lot and cease inspection.

Clearly, since the combined sample is subjected to the attributes test, the variables and attributes aspects of the plan are not independent. The probability of acceptance under this procedure becomes:

$$P_a = P(\bar{x} \leq A) + \sum_{i=0}^{c} \sum_{j=0}^{c-1} P_{n_1}(i, \bar{x} > A) \, P(j; n_2)$$

where

$P(\bar{x} \leq A)$ = probability that the sample mean does not exceed the acceptance limit A

$P(j; n_2)$ = probability of j defectives in a sample of size n_2

$P_{n_1}(i, \bar{x} > A)$ = joint probability of $\bar{x} > A$ and i defectives in a sample of size n_1.

Examination of the formula for the probability of acceptance, P_a, will indicate that all probabilities contained therein have been tabulated or can be calculated by elementary methods, with the exception of $P_{n_1}(i, \bar{x} > A)$. The same would be true for modified forms of the above procedure, say one having two acceptance numbers, c_1 and c_2, for the 2 samples. Thus, this joint probability is a very important factor in determining the properties of mixed sampling plans generally. It provides the key, and can be used directly, for determining the operating characteristic curves of mixed plans of the general type to be considered here -- for single specification limit, and known standard deviation.

2. METHOD OF EVALUATING $P_n(i, \bar{x} > A)$

2.1. Approach

This section will present a method of evaluating $P_n(i, \bar{x} > A)$, the probability that a sample of size n will have exactly i defectives and a sample mean $(\bar{x})$ greater than some limit A. First to be considered will be the case in which i = 0. This will be followed by the case in which i > 0. The method of evaluation used was suggested from the results of Gregory and Resnikoff [3] -- the solution for this probability when i = 0 is the same as that of their exposition[3].

In determining these probabilities, we will let $p_n(i, \bar{x})$ be the joint probability density of i and $\bar{x}$ in a sample of n. Then, given a particular value i = i_0, integration of this density over an appropriate region for $\bar{x}$ will give $P_n(i, \bar{x} > A)$ if the function is integrable and the integral exists.

2.2. Evaluation of $P_n(0, \bar{x} > A)$

Without loss of generality, since the z transformation will always be appropriate, consider random samples of n from the standard normal distribution, $\Phi(0,1)$. Choose z_U so that:

$$\frac{1}{\sqrt{2\pi}} \int_{z_U}^{\infty} e^{-\frac{t^2}{2}} \, dt = p \ .$$

Thus, z_U is an upper limit such that only 100p percent of the population exceeds the said limit. The probability density for samples with mean $\bar{z}$ and no observations greater than z_U is desired, that is, samples with mean $\bar{z}$ such that if the observations are ordered:

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[3]The Gregory and Resnikoff evaluation of a similar probability for i > 0 appears to be subject to question due to the particular restrictions placed on the region of integration.

$$-\infty < z_{(1)} \leq z_{(2)} \leq \cdots \leq z_{(n)} \leq z_U.$$

Now the probability that none of the observations in a random sample from $\phi(0,1)$ is a distance further from the sample mean than the limit z_U can be found from the distribution of the deviation of $z_{(n)}$ from the sample mean, $u_n = (z_{(n)}-\bar{z})$, requiring that $(z_{(n)}-\bar{z}) < (z_U-\bar{z})$. This distribution has been tabulated both by Nair [6] and Grubbs [7] as $F_n(u)$ for $\phi(0,1)$ where $F_n(u) = P(u_n \leq u)$ for a sample of size n.

It can be shown that the extreme deviate from the sample mean, u_n, and the sample mean $\bar{z}$ are independent. Consider the following theorem:

> Th. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution having a p.d.f. $f(x;\theta)$ $\gamma < \theta < \delta$. Let $Y_1 = u_1(X_1, X_2, \ldots, X_n)$ be a sufficient statistic for θ and let the p.d.f. $g_1(y_1;\theta), \gamma < \theta < \delta$, of Y_1 be complete. Let $Z = u(X_1, X_2, \ldots, X_n)$ be any other statistic (not a function of Y_1 alone). If the distribution of Z does not depend upon θ, then Z is stochastically independent of the sufficient statistic Y_1.[4]

Here $Y_1 = \bar{z}$ while $Z = u_n$. Also $\bar{z}$ is, for every $\sigma^2 > 0$, a sufficient statistic for $\mu, -\infty < \mu < \infty$, and the density function of the sufficient statistic $\bar{z}$ is complete. The cumulative distribution function of u_n is

$$F_n(u_n \leq u) = \frac{n\sqrt{n}}{\sqrt{n-1}} \int_0^u \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{n}{n-1}x^2)} F_{n-1}(\frac{n}{n-1}x)dx$$

with

$$F_2(u_n \leq u) = 2\sqrt{2} \int_0^u \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(2x^2)} dx$$

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[4] R. V. Hogg and A. T. Craig, Introduction to Mathematical Statistics (New York, McMillan, 1959) p. 123.

which does not contain μ and hence $u_n = (z_{(n)} - \bar{z})$ is stochastically independent of the sufficient statistic $\bar{z}$.

So when $i = 0$, the joint density for $\bar{z}$ in the range $-\infty < \bar{z} \leq z_U$ is

$$p_n(0, \bar{z}) = p(\bar{z}) \; P(z_{(n)} - \bar{z} < z_U - \bar{z}) = p(\bar{z}) \; F_n(z_U - \bar{z}).$$

But $p_n(0, \bar{z})$ is Riemann integrable in the region $-\infty < \bar{z} \leq z_U$ since it is the product of two continuous functions in the interval. For some value z_A such that $-\infty < z_A < z_U$,

$$P_n(0, \bar{z} > z_A) = \int_{z_A}^{z_U} \phi(0, \tfrac{1}{n}) \; F_n(z_U - \bar{z}) \; d\bar{z} =$$

$$\int_{z_A}^{z_U} \frac{\sqrt{n}}{\sqrt{2\pi}} \; e^{-\frac{n\bar{z}^2}{2}} \; F_n(z_U - \bar{z}) \; d\bar{z}$$

and the right hand side can be evaluated by numerical integration procedures.

2.3. Evaluation of $P_n(i, \bar{x} > A), i > 0$

Again without loss of generality, consider random samples from the standard normal distribution $\phi(0, 1)$. The probability density for samples with mean $\bar{z}$ and i observations greater than z_U is desired, that is, samples with mean $\bar{z}$ such that if the observations are ordered:

$$-\infty < z_{(1)} \leq z_{(2)} \leq \cdots \leq z_{(n-1)} \leq z_U < z_{(n-i+1)} \leq \cdots \leq z_{(n)} < \infty \; .$$

Suppose the sample to be selected sequentially. Further, suppose that the sequence of observations giving rise to such a sample was in the order indicated, that is, the first $(n-i)$ observations less than or equal to z_U and the last (i) observations

greater than z_U. Then the sample could be divided into two subsamples, the first $(n-1)$ values $\leq z_U$ and the second (1) values $> z_U$. If these subsamples are regarded separately, the means of the two samples would be related to the overall mean of the grand sample as:

$$(n-1)\bar{z}_1 + (1)\bar{z}_2 = (n)\bar{z}$$

$$\bar{z}_1 = \frac{n\bar{z} - 1\bar{z}_2}{n-1}$$

while clearly

$$-\infty < \bar{z}_1 \leq z_U$$

$$z_U < \bar{z}_2 < \infty.$$

Now to have a grand mean $\bar{z}$ and 1 defectives, there must be associated with $\bar{z}_1$, no sample value $> z_U$ and with $\bar{z}_2$ no sample value $\leq z_U$. But since the grand sample has been divided into two subsamples, each of them can be treated in exactly the manner of the above section for $p_n(0,\bar{z})$. So for $\bar{z}_1$:

$$p_{n-1}(0,\bar{z}_1) = p(\bar{z}_1)F_{n-1}(z_U-\bar{z}_1).$$

And treating the lower limit on the second sample as if it were an upper limit:

$$p_1(1,\bar{z}_2) = p(\bar{z}_2)F_1(\bar{z}_2-z_U)$$

since $F_n(u) = P(u_n \leq u) = P(-u \leq u_1)$.

Note that in the above section it was shown that $(z_{(n)}-\bar{z})$ is independent of $\bar{z}$ for the sample. Since each of the subsamples is being treated as an entity in itself, the magnitude of $\bar{z}_1$ will not affect $F_n(u)$. This, of course, must be the case for the statistic $(\frac{x_{(n)}-\bar{x}}{\sigma})$ to be useful in testing outliers, its principal application.

For a grand mean $\bar{z}$ in the range $-\infty < \bar{z} < \infty$ and $\bar{z}_1$ and $\bar{z}_2$ related as shown above, the joint density is such that

$$p_n(i,\bar{z}) = \binom{n}{i} \, p_{n-i}(0,\bar{z}_1)p_i(1,\bar{z}_2)$$

$$= \binom{n}{i}p(\bar{z}_1)F_{n-i}(z_U-\bar{z}_1)p(\bar{z}_2)F_i(\bar{z}_2-z_U)$$

since there are $\binom{n}{i}$ possible combinations of the observations to give $n-i \leq U$ and $i > U$.

Now $p_n(i,\bar{z})$ certainly does not define a continuous surface. However, for a fixed value of $i = i_0$, it becomes a product of continuous functions and is therefore Riemann integrable over an appropriate finite range. Furthermore, it is dominated by an appropriate multiple of the normal density and therefore the improper integral exists. Again for any specified value z_A, $-\infty < z_A < \infty$, $p_n(i,\bar{z})$ must be integrated over a suitable region to obtain the desired probability, $P_n(i,\bar{z} > z_A)$. Given $\bar{z}$, if $\bar{z}_2$ is allowed to vary so that $z_U < \bar{z}_2 < \infty$, then $\bar{z}_1$ will be determined by the relationship:

$$\frac{n\bar{z} - i\bar{z}_2}{n-i} < \bar{z}_1 \leq z_U.$$

For $P_n(i,\bar{z} > z_A)$, however, $\bar{z}$ is never $< z_A$, z_A being the greatest lower bound for the interval,

so

$$\frac{nz_A-i\bar{z}_2}{n-i} < \bar{z}_1 \leq z_A.$$

Hence, for $i = i_0 > 0$ and appropriate regions of integration for $\bar{z}_1$ and $\bar{z}_2$, $R\bar{z}_1$ and $R\bar{z}_2$ respectively:

$$P_n(1,\bar{z} > z_A) = \int\limits_{R\bar{z}_2} \int\limits_{R\bar{z}_1} \binom{n}{1} p(\bar{z}_2) F_1(\bar{z}_2 - z_U) p(\bar{z}_1) F_{n-1}(z_U - \bar{z}_1) d\bar{z}_1 d\bar{z}_2$$

$$= \binom{n}{1} \int\limits_{R\bar{z}_2} \int\limits_{R\bar{z}_1} \frac{\sqrt{1}}{\sqrt{2\pi}} e^{-\frac{1}{2}\bar{z}_2^2} F_1(\bar{z}_2 - z_U) \frac{\sqrt{n-1}}{\sqrt{2\pi}} e^{-\frac{(n-1)}{2}\bar{z}_1^2} F_{n-1}(z_U - \bar{z}_1) d\bar{z}_1 d\bar{z}_2$$

$$= \binom{n}{1} \int\limits_{z_U}^{\infty} \int\limits_{\frac{nz_A - 1\bar{z}_2}{n-1}}^{z_U} \frac{\sqrt{1(n-1)}}{2\pi} e^{-\frac{1}{2}\left[1\bar{z}_2^2 + (n-1)\bar{z}_1^2\right]} F_1(\bar{z}_2 - z_U) F_{n-1}(z_U - \bar{z}_1) d\bar{z}_1 d\bar{z}_2$$

which can be integrated numerically as will be shown.

2.4. Summary

The probability $P_n(1,\bar{z} > z_A)$ can be evaluated by use of the formulas:

$$P_n(0,\bar{z} > z_A) = \int\limits_{z_A}^{z_U} \frac{\sqrt{n}}{\sqrt{2\pi}} e^{-\frac{n\bar{z}^2}{2}} F_n(z_U - \bar{z}) d\bar{z} , \qquad 1 = 0$$

and, for $1 = 1_0 > 0$

$$P_n(1,\bar{z} > z_A) = \binom{n}{1} \int\limits_{z_U}^{\infty} \int\limits_{\frac{nz_A - 1\bar{z}_2}{n-1}}^{z_U} \frac{\sqrt{1(n-1)}}{2\pi} e^{-\frac{1}{2}\left[1\bar{z}_2^2 + (n-1)\bar{z}_1^2\right]}$$

$$F_1(\bar{z}_2 - z_U) F_{n-1}(z_U - \bar{z}_1) d\bar{z}_1 d\bar{z}_2 , \quad 1 > 0$$

$P_n(0,\bar{z} > z_A)$ and $P_n(1,\bar{z} > z_A)$, then, represent values of the probability desired for samples taken from a standard normal universe. Using the z transformation for a particular $1 = 1_0$:

$$P_n(1,\bar{x} > A) = P_n(1,\bar{z} = \frac{\bar{x}-\mu}{\sigma} > s_A = \frac{A-\mu}{\sigma})$$

and conversely

$$P_n(1,\bar{z} > s_A) = P_n(1,\bar{x} = \bar{z}\sigma + \mu > A = s_A\sigma + \mu)$$

3. COMPUTATION OF $P_n(1,\bar{x} > A)$

3.1. Scheme for Computation

The computation of

$$P_n(0,\bar{z} > z_A) = \int_{z_A}^{z_U} \frac{\sqrt{n}}{\sqrt{2\pi}} e^{-\frac{n\bar{z}^2}{2}} F_n(z_U-\bar{z})d\bar{z}$$

can be accomplished by the application of appropriate methods of numerical integration to the integrand between the limits indicated. Values of $F_n(z_U-\bar{z})$ may be obtained by interpolation from the tables of $F_n(u)$ of both Nair [6] and Grubbs [7] to arrive at the functional values.

For a particular $i = i_0$, the computation of

$$P_n(i,\bar{z} > z_A) = \binom{n}{1} \int_{z_U}^{\infty} \int_{\frac{nz_A-i\bar{z}_2}{n-i}}^{z_U} \frac{\sqrt{i(n-i)}}{2\pi} e^{-\frac{1}{2}\left[i\bar{z}_2^2+(n-i)\bar{z}_1^2\right]}$$

$$F_i(\bar{z}_2-z_U)F_{n-i}(z_U-\bar{z}_1)d\bar{z}_1 d\bar{z}_2$$

is greatly simplified by rewriting the integral as

$$P_n(1, \bar{z} > z_{\Lambda}) = \binom{n}{1} \int_{z_U}^{\infty} \frac{\sqrt{1}}{\sqrt{2\pi}} \bullet\, e^{-\frac{1}{2}\bar{z}_2^2}\, F_1(\bar{z}_2 - z_U)$$

$$\left[\frac{\sqrt{n-1}}{\sqrt{2\pi}} \int_{\frac{nz_{\Lambda}-1\bar{z}_2}{n-1}}^{z_U} e^{-\frac{(n-1)\bar{z}_1^2}{2}}\, F_{n-1}(z_U - \bar{z}_1)\, d\bar{z}_1 \right]\, d\bar{z}_2$$

and noting that

$$P_{n-1}\left(0, \bar{z} > \frac{nz_{\Lambda}-1\bar{z}_2}{n-1}\right) = \int_{\frac{nz_{\Lambda}-1\bar{z}_2}{n-1}}^{z_U} \frac{\sqrt{n-1}}{\sqrt{2\pi}} \bullet\, e^{-\frac{(n-1)\bar{z}^2}{2}}\, F_{n-1}(z_U - \bar{z})\, d\bar{z}$$

when calculated for a sample of size (n-1). This is exactly the expression which appears in brackets in the formula for $P(1, \bar{z} > z_{\Lambda})$ above.

So,

$$P(1, \bar{z} > \bar{z}_{\Lambda}) = \binom{n}{1} \int_{z_U}^{\infty} \frac{\sqrt{1}}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\bar{z}_2^2}\, F_1(\bar{z}_2 - z_U) \left[P_{n-1}\left(0, \bar{z}_1 > \frac{nz_{\Lambda}-1\bar{z}_2}{n-1}\right) \right]\, d\bar{z}_2$$

Treating the integrand as one function for purpose of numerical integration it is possible to interpolate for values of $P_{n-1}\left(0, \bar{z}_1 > \frac{nz_{\Lambda}-1\bar{z}_2}{n-1}\right)$ once tables of this probability are available. In the same way $F_1(\bar{z}_2 - z_U)$ may be obtained from existing tables, where available, to arrive at values of the integrand.

In evaluating the integral expression for $P_n(1, \bar{z} > z_{\Lambda})$ it is worthwhile to note that the values of 1 for which $F_1(\bar{z}_2 - z_U)$ is calculated will generally be small. But $F_1(u) = 1$ for all values of u, since, for n = 1, it represents $P(x_1 - x_1 \leq u)$ for

$u \geq 0$. Further, for i = 2:

$$F_2(u) = 2\sqrt{2} \int_0^u \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(2x^2)} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^u e^{-x^2} dx$$

which is simply the error function for which standard approximations are available. These relationships simplify the computation of $P_n(i,\bar{z} > s_A)$ still further.

5. CONCLUSION

This report presents a method for evaluating joint probabilities necessary to determine the operating characteristics of dependent mixed plans for the case of single specification limit, known standard deviation, assuming an underlying normal distribution. It also provides a technique for facilitating the computation of these joint probabilities. Such values will be of substantial aid to any comprehensive studies that may be made to determine the best composite structure and the relative merits of dependent mixed plans, or to explore the possibility of developing suitable sets of mixed plans for practical application. Tables of values for a number of small sample sizes are being developed.

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13. ABSTRACT

This report presents a method of evaluating certain joint probabilities necessary to determine the operating characteristics of dependent mixed (variables and attributes) sampling inspection plans for the case of single specification limit, known standard deviation, and normal distribution. It also provides a technique for facilitating the computation of these joint probabilities.

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14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.